DISTRIBUTED FORCES

When forces are continuously distributed over a region of a structure, the cumulative effect of this distribution must be determined. The designers of high-performance sailboats consider both air-pressure distributions on the sails and water-pressure distributions on the hull.

5 DISTRIBUTED FORCES

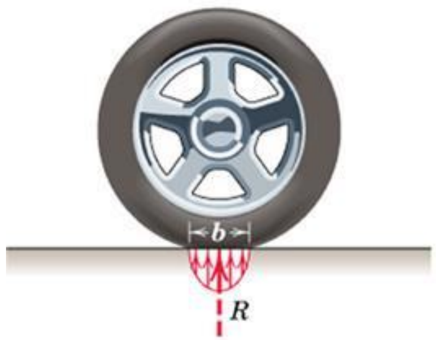
CHAPTER OUTLINE

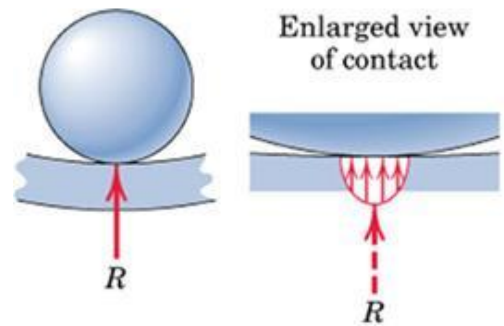
5/1 INTRODUCTION

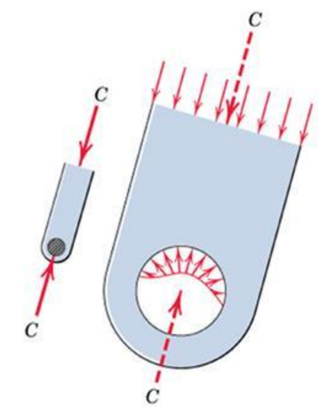
- In the previous chapters we treated all forces as concentrated along their lines of action and at their points of application.
- This treatment provided a reasonable model for those forces. Actually, "concentrated" forces do not exist in the exact sense, since every external force applied mechanically to a body is distributed over a finite contact area, however small.

DISTRIBUTED FORCES

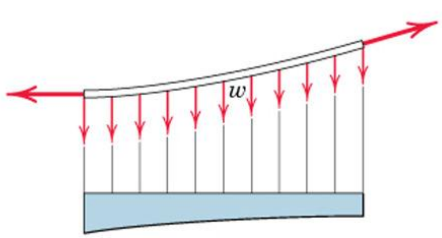
Concentrated Forces: If dimension of the contact area is negligible compared to other dimensions of the body the contact forces may be treated as Concentrated Forces

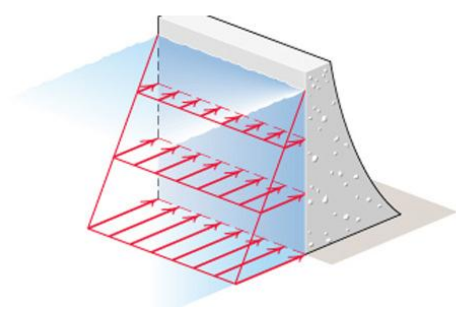


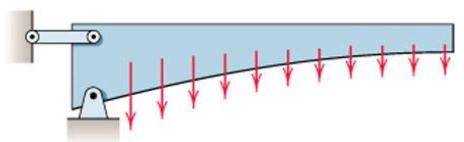




Distributed Forces: If forces are applied over a region whose dimension is not negligible compared with other pertinent dimensions proper distribution of contact forces must be accounted for to know intensity of force at any location.



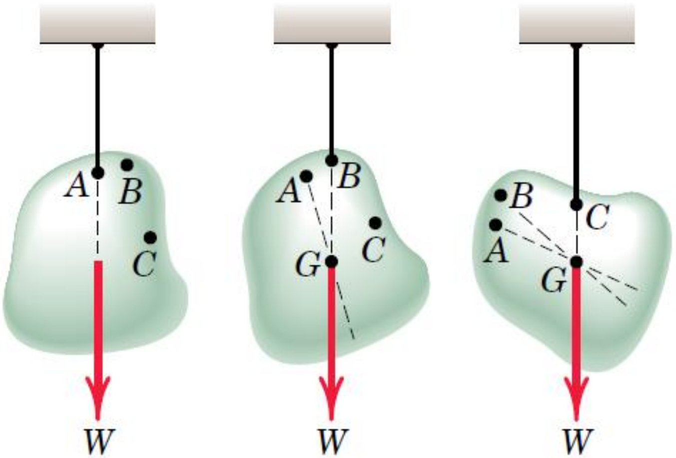




DISTRIBUTED FORCES

SECTION A CENTERS OF MASS AND CENTROIDS

5/2 CENTER OF MASS

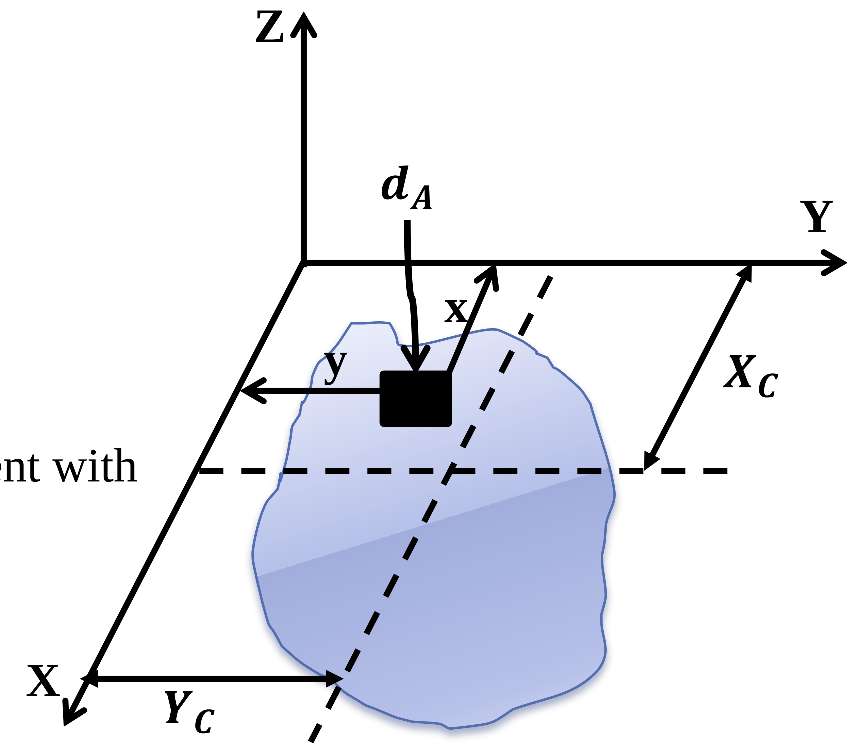


(*a*) (*b*) (*c*)

- If we suspend the body, as shown in Fig. 5/3, from any point such as ***A***, the body will be in equilibrium under the action of the tension in the cord and the resultant ***W*** of the gravitational forces acting on all particles of the body.
- This resultant is clearly collinear with the cord.
- Dotted lines show lines of action of the resultant force in each case.
- These lines of action will be concurrent at a single point ***G***, which is called the *center of gravity* of the body.

Centroid of an Area

The point located by the coordinates X_c and Y_c is defined as the centroid of the area. The centroid must lie in the plane of the area and two coordinates are sufficient to locate it by the following procedure



1. Take an element wit area (dA).
2. Find the total area of the body (A).

$$A = \int d_A \text{ ....}(5.1)$$

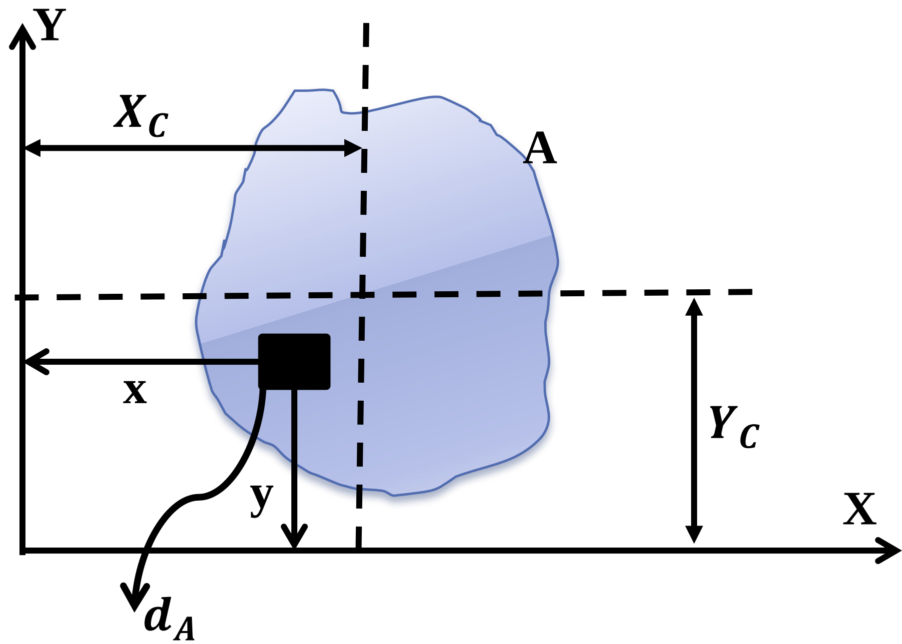
3. Determine the first moment of the element with respect X-axis and Y-axis.

DISTRIBUTED FORCES

4-

$$\left.\begin{aligned} dM_x &= y.\,d_A \\ dM_y &= X.\,d_A \end{aligned}\right\} \quad \text{..........} \; 5.2$$

$$\left.\begin{aligned} M_x &= \int y.\,d_A \\ M_y &= \int x.\,d_A \end{aligned}\right\} \quad \text{..........} \; 5.3$$



Where

x = is the distance from the centroid of the element to the y-axis.

y = is the distance from the centroid of the element to the y-axis

5. Find the coordinates of the centroid of the body as follows:-

$$\left.\begin{aligned} X_c &= \frac{\int x\, dA}{\int dA} = \frac{M_y}{A} \\ y_c &= \frac{\int y\, dA}{\int dA} = \frac{M_x}{A} \end{aligned}\right\} \quad \ldots\ldots\ldots\ 5.4$$

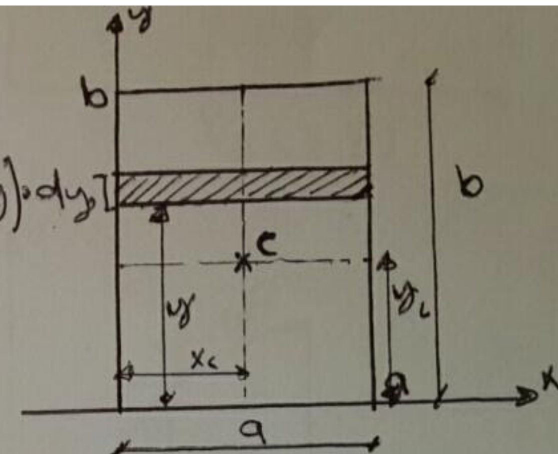
The first moment of an area about on axis has units of mm^3, cm^3, m^3. the area is s scalar quantity and the sense of moment of an area depends on the moment arm, so, its positive if the area is on one side of the opposite side if the axis (see fig 5-2 b, c)

DISTRIBUTED FORCES

Ex:- find the centroid of rectangle has a, b dimensions, as shown in figure below.

Solution

① To find y_c, take element with depth (dy), dy]



$$y_c = \frac{\int y \cdot dA}{\int dA} = \frac{M_x}{A}$$

$$A = \int dA = \int_0^b a \cdot dy = a \cdot y \Big|_0^b$$

$$A = ab - a \cdot 0 \Rightarrow A = a \cdot b$$

$$M_x = \int_0^b y \cdot dA = \int_0^b y\,(a\,dy) = \frac{a y^2}{2}\Big|_0^b \Rightarrow M_x = \frac{ab^2}{2}$$

$$\therefore y_c = \frac{M_x}{A} = \frac{ab^2/2}{a.b} = \frac{b}{2}$$

DISTRIBUTED FORCES

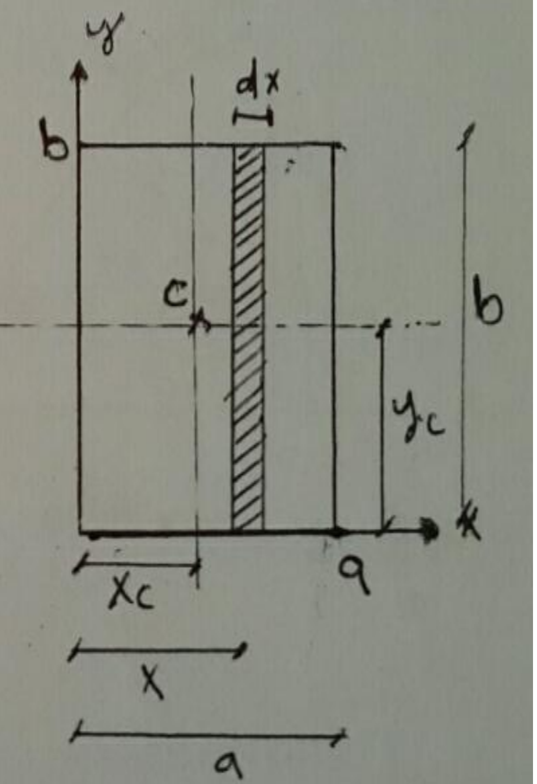
② To find X_c, take element with respect width (dx)

$$X_c = \frac{\int x \cdot dA}{\int A} = \frac{M_y}{A}$$

$$M_y = \int_0^a x \, dA = \int_0^a x\,(b \cdot dx) = \frac{bx^2}{2}\Big|_0^a$$

$$M_y = a^2 b/2$$

$$X_c = \frac{M_y}{A} = \frac{a^2 b/2}{a \cdot b} = \frac{a}{2}$$



DISTRIBUTED FORCES

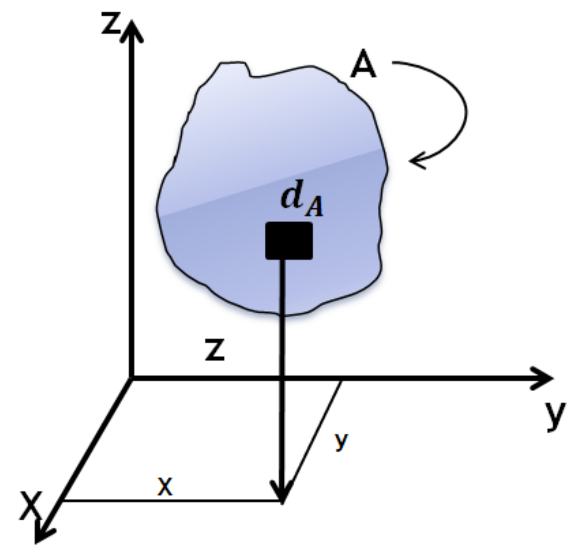
General Manners

For 3D

$$X_c = \frac{\int x \, dA}{\int dA} = \frac{M_{yz}}{A}$$

$$y_c = \frac{\int y \, dA}{\int dA} = \frac{M_{xz}}{A}$$

$$z_c = \frac{\int z \, dA}{\int dA} = \frac{M_{xy}}{A}$$

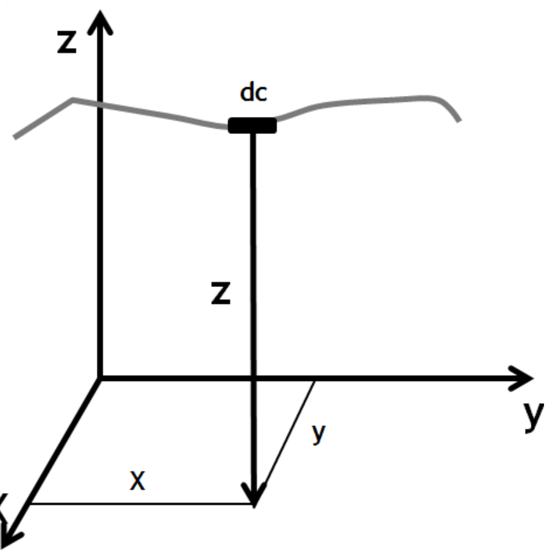


Centroid of curve surface centroid of the

$$X_c = \frac{\int x \, dL}{\int dL}$$

$$y_c = \frac{\int y \, dL}{\int dL}$$

$$z_c = \frac{\int z \, dL}{\int dL}$$

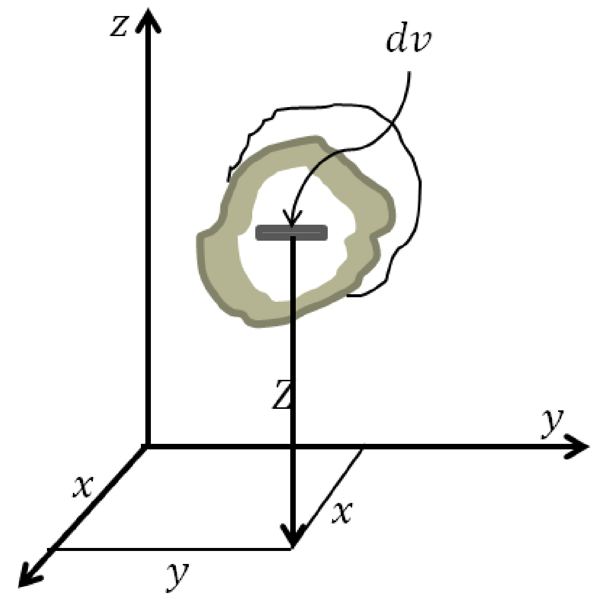


DISTRIBUTED FORCES

Centroid of volume

$$X_c = \frac{\int x\, dv}{\int dv} \qquad y_c = \frac{\int y\, dv}{\int dv}$$

$$z_c = \frac{\int z\, dv}{\int dv}$$



Center of Mass

$$\boldsymbol{dm = \rho . dv}$$

$$X_m = \frac{\int x\, dm}{\int dm} = \frac{\int x\, \rho . dv}{\int \rho . dv}$$

$$y_m = \frac{\int y\, dm}{\int dm} = \frac{\int y\, \rho . dv}{\int \rho . dv}$$

$$z_m = \frac{\int z\, dm}{\int dm} = \frac{\int z\, \rho . dv}{\int \rho . dv}$$

DISTRIBUTED FORCES

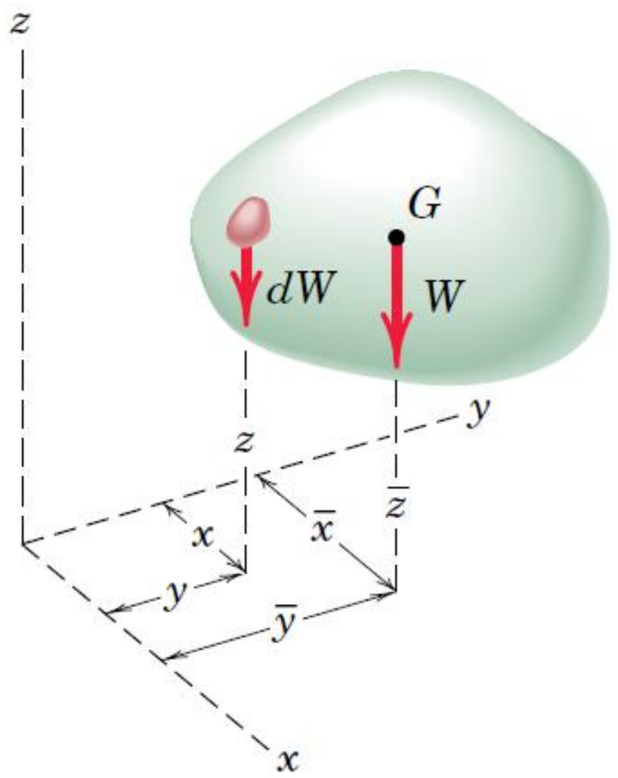
Center of gravity

$$dw = gdm = \rho . g . dv$$

$$X_w = \frac{\int x\, dw}{\int dw} = \frac{\int x\, \rho . g . dv}{\int \rho . dv}$$

$$y_w = \frac{\int y\, dw}{\int dw} = \frac{\int y\, \rho . g . dv}{\int \rho . dv}$$

$$z_w = \frac{\int z\, dw}{\int dw} = \frac{\int z\, \rho . g . dv}{\int \rho . dv}$$



DISTRIBUTED FORCES

Ex:- determine the coordinate of the centroid of the circular arc(line), in figure below.

* from Symmetry, the Centroid must lie on a line through the origion at an angle $\alpha/2$ with x-axis.

* The length of the element is

$$dL = r\,d\theta$$

$$\therefore L = \int dL = \int_0^{\alpha} r\,d\theta = r\,\theta\Big|_0^{\alpha}$$

$$\therefore L = r\alpha$$

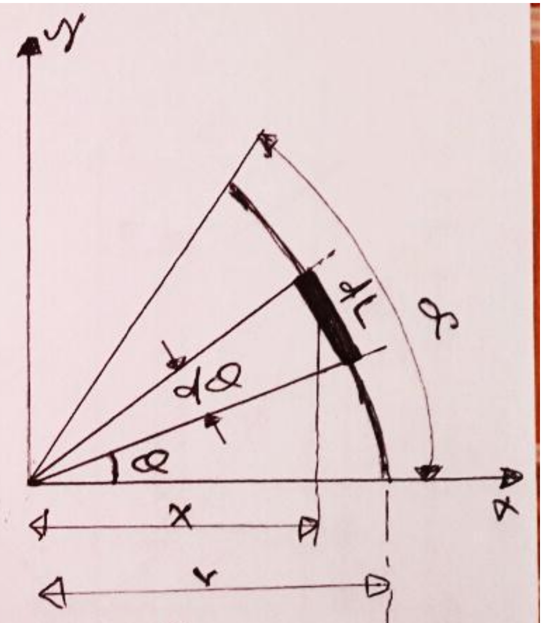


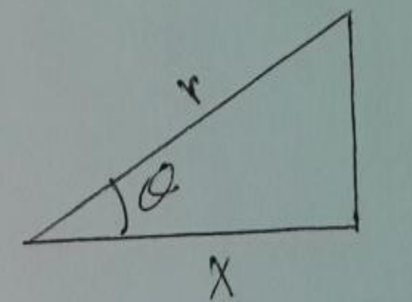
fig (7-9)

$$dM_y = x \cdot dL = (r\cos\theta)(r\,d\theta) = r^2\cos\theta\,d\theta$$

$$M_y = \int_0^{\alpha} r^2\cos\theta\,d\theta = r^2\sin\Big|_0^{\alpha} \Rightarrow \underline{M_y = r^2\sin\alpha}$$

DISTRIBUTED FORCES

$$x_c = \frac{M_y}{L} = \frac{r^2 \sin\alpha}{r\alpha} = \frac{r\sin\alpha}{\alpha}$$



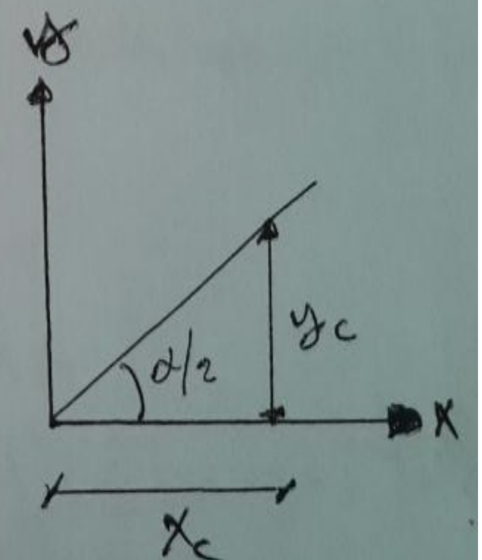
$x = r\cos\theta$

$$\tan\frac{\alpha}{2} = \frac{y_c}{x_c}$$

$$\therefore \quad y_c = x_c \cdot \tan\frac{\alpha}{2} \Rightarrow \boxed{y_c = \frac{r\sin\alpha}{\alpha}\cdot\tan\frac{\alpha}{2}}$$

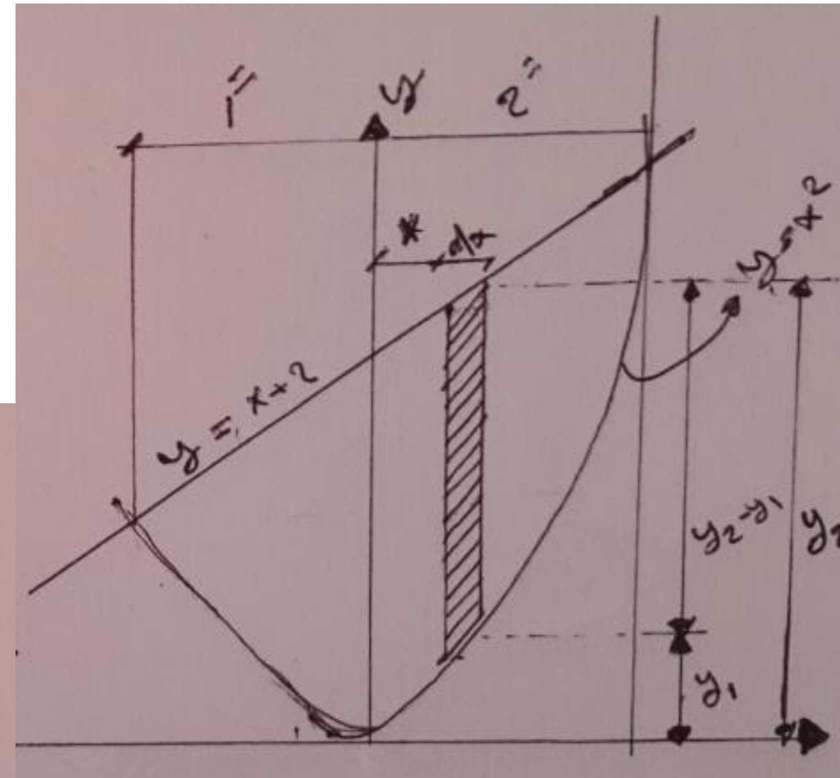
if $\alpha = \frac{\pi}{2} \Rightarrow \frac{\alpha}{2} = \frac{\pi}{4}$

$$x_c = y_c = \frac{2r}{\pi}$$



DISTRIBUTED FORCES

Ex:- Determine the coordinate of the centroid of the area in figure below, bounded by the curve $y = x^2$ and the line $y = x + 2$. in the equations $x\ and\ y$ are in inches.



Solution:-

① $dA = (y_2 - y_1)\,dx$

$dA = (x - 2 - x^2)\,dx$

$\therefore A = \int_{-1}^{2} (x+2-x^2)\,dx$

$= \frac{x^2}{2} + 2a - \frac{x^3}{3} \Big|_{-1}^{2} \Rightarrow A = 4.5\ cm^2$

DISTRIBUTED FORCES

② The moment of element with respect to y-axis — fig

$$dM_y = x \cdot dA = x\,(x+2-x^2)\,dx = (x^2+2x-x^3)\,dx$$

$$M_y = \int_{-1}^{2}(x^2+2x-x^3)\,dx = \left[\frac{x^3}{3}+x^2-\frac{x^4}{4}\right]_{-1}^{2}$$

$$M_y = 2.25\ \text{in}^3$$

$$\therefore\ x_c = \frac{M_y}{A} = \frac{2.25}{4.5} = 0.5\ \text{in}$$

(3) The location of the centroid of the element with respect to xy-plan.

$$\left(y + \frac{y_2 - y_1}{2}\right) = \left(\frac{y_1 + y_2}{2}\right)$$

$$dM_x = \left(\frac{y_1 + y_2}{2}\right) dA = \frac{1}{2}(y_1 + y_2) \cdot dA$$

$$dM_x = \frac{1}{2}\left(x^2 + 4x + 4 - x^4\right) dx \Rightarrow M_x = \frac{1}{2}\int_{-1}^{2}\left(x^2 + 4x + 4 - x^4\right) dx$$

$$M_x = \frac{1}{2}\left[\frac{x^3}{3} + 2x^2 + 4x - \frac{x^5}{5}\right]_{-1}^{2} \Rightarrow M_x = 7.20 \ in^3$$

$$y_c \ \frac{M_c}{A} = \frac{7.20 \ in^3}{4.5 \ cm} = 1.6 \ in$$